Secure Two-party Computation

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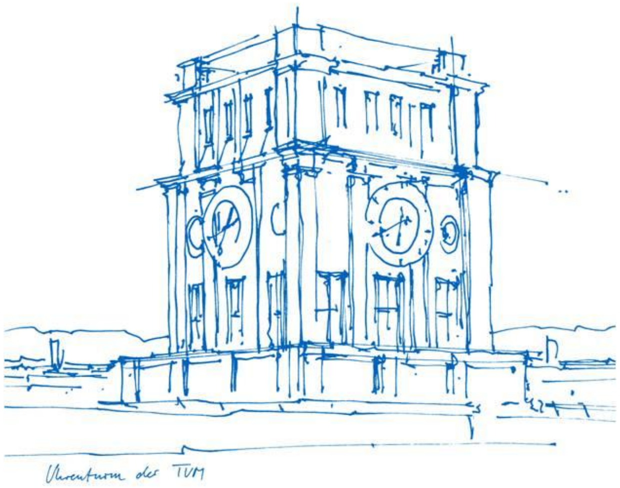


What is secure computation?

Collaboratively compute a function and maintain input secrecy

- **Multi-party Computation (MPC)**: MPC is the joint computation of a public function $f(\mathbf{x})=y$ between M parties with input x_M such that no party learns the private inputs of the counterparties
  - The **adversary** is assumed to corrupt T parties.
- **Secure two-party computation (2PC)**: Secure 2PC uses M=2 and T=1 such that two parties with private inputs x_1, x_2 can collaboratively compute $f(x_1, x_2)$ without learning any other x_i

Adversarial behavior model

Assumption

- **Semi-honest parties** honestly follow the protocol specification
  - Attack/Try to learn from exchanged parameters

- **Malicious adversaries** arbitrarily deviate from the protocol specification
  - Inject false values such that the opponent accepts values without notice
  - Selective-failure attack: Inject false values, then observe and learn from failure
    - (e.g. know secret permutation & corrupt a row, learn which row was evaluated, learn information on inputs)

MPC and cryptographic building blocks

Constructions

- **Garbled circuits**
  - Based on oblivious transfer (OT), OT cost depends on input size
  - Two-party computation
  - Constant communication (independent of circuit depth)

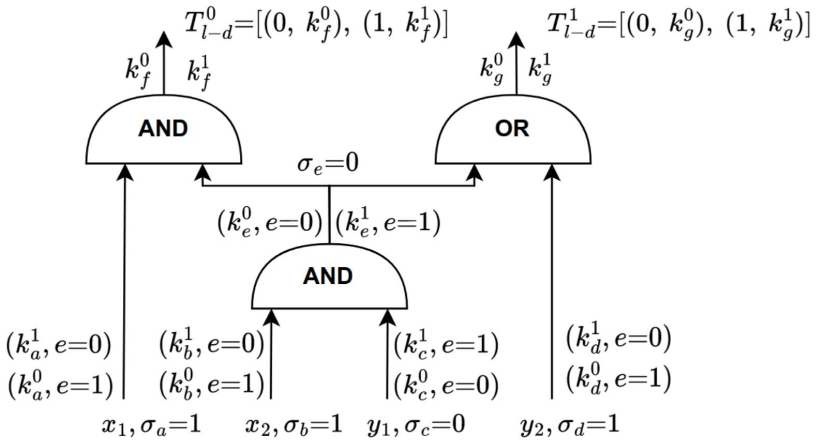
- **GMW, BGW, CCD**
  - Based on secret sharing
  - Multi-party computation
  - Boolean (AND, OR) or arithmetic (MUL, ADD) circuits
  - Gate-by-gate computation in multiple oblivious communication rounds for MUL operation

SoK: Compilers for MPC
https://www.cis.upenn.edu/~stevez/papers/HHNZ19.pdf

2PC based on garbled circuits

Garbled circuit parameters

- **Parties**: Garbler and evaluator
- **Boolean circuit**: AND, OR gates
- **Wire labels**: k, i, e, sigma
- **Private labels**: i (internal), sigma
- **Public labels**: e (external), k^i
- **Random labels**: sigma, k^i



	$G(\mathcal{C}^{0,(1,2)}_{\text{AND}})$
0,0	$E_{k^1_b}(E_{k^0_c}(k^0_e \| 0))$
0,1	$E_{k^1_b}(E_{k^1_c}(k^1_e \| 1))$
1,0	$E_{k^0_b}(E_{k^0_c}(k^0_e \| 0))$
1,1	$E_{k^0_b}(E_{k^1_c}(k^0_e \| 0))$

	$G(\mathcal{C}^{1,(0,1)}_{\text{AND}})$
0,0	$E_{k^1_a}(E_{k^0_e}(k^0_f))$
0,1	$E_{k^1_a}(E_{k^1_e}(k^1_f))$
1,0	$E_{k^0_a}(E_{k^0_e}(k^0_f))$
1,1	$E_{k^0_a}(E_{k^1_e}(k^0_f))$

	$G(\mathcal{C}^{1,(1,3)}_{\text{OR}})$
0,0	$E_{k^0_e}(E_{k^1_d}(k^1_g))$
0,1	$E_{k^0_e}(E_{k^0_d}(k^0_g))$
1,0	$E_{k^1_e}(E_{k^1_d}(k^1_g))$
1,1	$E_{k^1_e}(E_{k^0_d}(k^1_g))$

External labels e

Protocol to evaluate semi-honest 2PC circuit

Example: Garbler input **x**=[x_1=1,x_2=0], Evaluator input **y**=[y_1=0,y_2=1], $\sigma_a=1,\sigma_b=1,\sigma_c=0,\sigma_d=1$

1. **Garbler garbles**: sample sigmas, k_L^i with L ∈ {a,b,c,d,e,f,g} (16B with aes128), compute e_L = sigma xor i, compute G tables, send (**G,** circuit C, (k^1_a, e=0), (k^0_b, e=1), $T_{l\text{-}d}$)

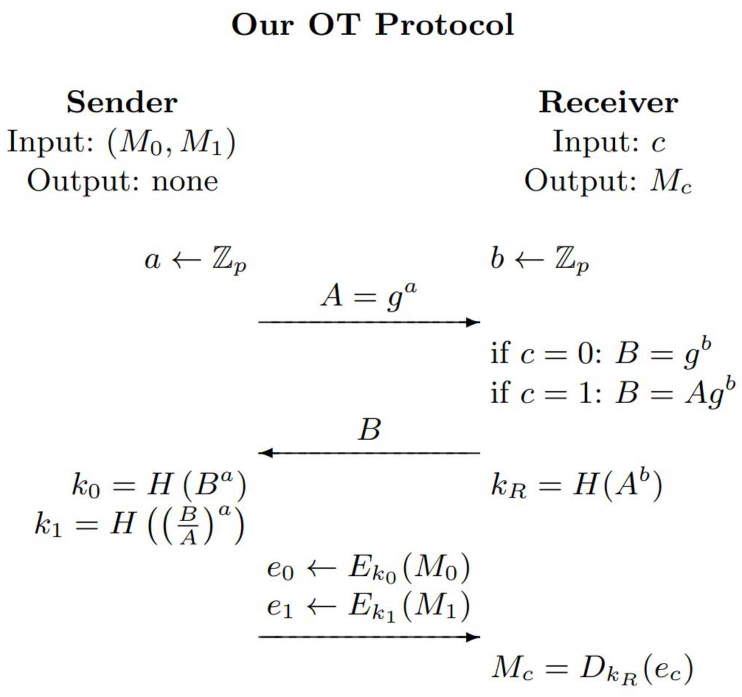
2. **Garbler & Evaluator using OT**: for every input wire corresponding to y input bit, run
   1-out-of-2 OT with (m_1=[k^0_L, e], m_2=[k^1_L, e]).

   $OT_{c,d}$ with b_c=0, b_d=1 yields (k^0_c, e=0), (k^1_d, e=0)

Oblivious Transfer

Transfer 2 messages without learning which message is picked

- 1-out-of-2 OT with 2 messages k_0, k_1
- Evaluator obtains (k_b ,e), with b ∈ {0,1}



The simplest protocol for OT:
https://eprint.iacr.org/2015/267.pdf

Protocol to evaluate semi-honest 2PC circuit

Example: Garbler input **x**=[x_1=1,x_2=0], Evaluator input **y**=[y_1=0,y_2=1], σ_a=1,σ_b=1,σ_c=0,σ_d=1

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   $OT_{c,d}$ with b_c=0, b_d=1 yields (k^0_c, e=0), (k^1_d, e=0)

3. **Evaluator use G tables to evaluate**

   Use G_{AND0} and (k^0_b, e=1), (k^0_c, e=0) -> row (1,0) to obtain (k^0_e, e=0)

   Use G_{AND1} and (k^1_a, e=0), (k^0_e, e=0) -> row (0,0) to obtain (k^0_f)

   Use G_{OR1} and (k^0_e, e=0), (k^1_d, e=0) -> row (0,0) to obtain (k^1_g)

   Use $T_{l\text{-}d}$ map to obtain out_0=0 from k^0_f and out_1=1 from k^1_g

Protocol to evaluate semi-honest 2PC circuit

Example: Garbler input $\mathbf{x}=[x_1=1,x_2=0]$, Evaluator input $\mathbf{y}=[y_1=0,y_2=1]$, $\sigma_a=1,\sigma_b=1,\sigma_c=0,\sigma_d=1$

3. **Evaluator use G tables to evaluate**

   …

   Use $T_{l\text{-}d}$ map to obtain $out_0=0$ from k^0_f and $out_1=1$ from k^1_g

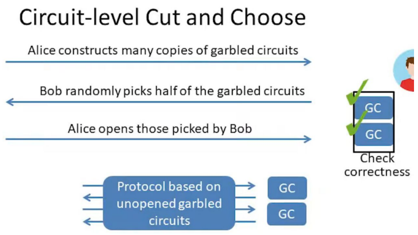
4. **Share output back to garbler**

   With ($out_0=0$, $out_1=1$) garbler knows wire keys at output labels

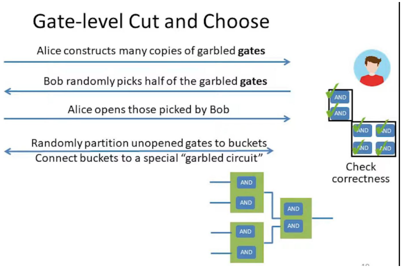
   However, ambiguity of wire keys in G_{AND1} together with OT obfuscates input of evaluator

From semi-honest to maliciously secure 2PC

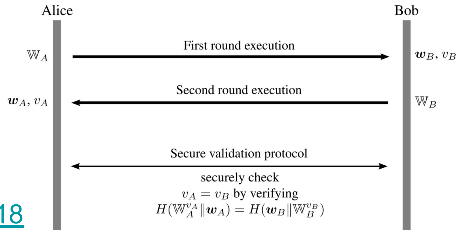
Techniques to secure 2PC against malicious adversaries



- **Cut-and-choose**: many copies of garbled circuits, validate random subset, use unopened circuits
  - Exist at a circuit and gate level



- **Dual execution**: two rounds semi-honest 2pc + secure validation
- **Authenticated garbling**: malicious secret sharing of GC permutation bits
  - Based on TinyOT protocol



Xiao Wang: Authenticated Garbling for Efficient Maliciously Secure
https://www.youtube.com/watch?v=8zCqki-ilZM

Quid-Pro-Quo-tocols
https://ieeexplore.ieee.org/abstract/document/6234418

Code example

MPC repository

Thank You

Questions?